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Characterizing nilpotent n-Lie algebras by their multiplier, $t(A) = 11, 12$

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Abstract

For every d-dimensional nilpotent n-Lie algebra A, $t(A)$ is defined by $t(A) = \binom{d}{n} - \dim \mathcal{M}(A)$, where $\mathcal{M}(A)$ denotes the Schur multiplier of A. In this paper, we classify all nilpotent n-Lie alegbras A satisfying $t(A) = 11, 12$. We also classify all nilpotent n- Lie algebras for $t(A) = 17, 18$, where $n \geq 3$.

Keywords: n-Lie algebra, nilpotent n-Lie algebra, multiplier
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1 Introduction

In 1985, Fillipov [10] introduced the concept of n-Lie algebras as an n-ary multilinear and skew-symmetric operation $[x_1, x_2, \dots, x_n]$, which satisfies the following generalized Jacobi identity

$$[[x_1, x_2, \dots, x_n], y_2, \dots, y_n] = \sum_{i=1}^{n} [x_1, \dots, x_{i-1}, [x_i, y_2, \dots, y_n], x_{i+1}, \dots, x_n]$$

This algebra becomes a ordinary Lie algebra when $n = 2$.

The Schur multiplier of n-Lie algebra A, denoted by $\mathcal{M}(A)$, is defined as $\mathcal{M}(A) = R \cap F^2/[R, F, \dots, F]$, where $A \cong F/R$ and F is a free n-Lie algebra. (see [5, 8] for more information on the Schur multiplier of n-Lie algebras).

Assume that $A_1, \dots, A_n$ are subalgebras of A. Then the subspace of A generated by all vectors $[x_1, \dots, x_n]$ where $x_i \in A_i$ is denoted by $[A_1, \dots, A_n]$. The subalgebra $A^2 = [A, \dots, A]$ is called the derived subalgebra of A. We say d-dimensional n-Lie algebra A is abelian and denoted by $F(d)$ if $A^2 = 0$. The center of A is defined as

$$Z(A) = \{x \in A : [x, A, \dots, A] = 0\}.$$

An n-Lie algebra A is called nilpotent if $A^s = 0$ for some non-negatve integer s, where A^{i+1} defines inductively as $A^1 = 1$ and $A^{i+1} = [A^i, A, \dots, A]$ for $i \geq 1$ (see [14, 16] for more details).

Eshrati et al. [8] showed for every d-dimensional n-Lie algebra A, there exists a non-negative integer $t(A)$ such that

$$\dim \mathcal{M}(A) + t(A) = \binom{d}{n}.$$

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All finite dimensional nilpotent Lie algebras A with $0 \leq t(A) \leq 10$ have been classified by several papers [1, 3, 12, 13]. Also all finite dimensional nilpotent n-Lie algebras A with $0 \leq t(A) \leq 16$ have been classified in [3, 6].

In this paper, we classify nilpotent Lie algebras A satisfying $t(A) = 11, 12$. We also classify nilpotent n-Lie algebras A satisfying $t(A) = 17, 18$ where $n \geq 3$. All n-Lie algebras are finite dimensional, and every non-mentioned bracket is assumed to be zero. Special Heisenberg n-Lie algebras play important roles in classification of n-Lie algebras. These n-Lie algebras are introduced in [8]. An n-Lie algebra A is called special Heisenberg if $A^2 = Z(A)$ and $\dim A^2 = 1$. Every special Heisenberg n-Lie algebra has dimension $mn+1$, where m is a natural number. An special Heisenberg n-Lie algebra of dimension $mn+1$ is given by

$$H(n,m) = \langle x, x_1, \ldots, x_{mn} : [x_{n(i-1)+1}, x_{n(i-1)+2}, \ldots, x_{ni}] = x, i = 1, \ldots, m \rangle.$$

The dimension of Schur multiplier of $F(d)$ and special Heisenberg n-Lie algebra are computed in Theorem 3.4 of [5] and Theorem 2.3 of [8] as follows.

Theorem 1.1. *We have* $\dim \mathcal{M}(F(d)) = \binom{d}{n}$, $\dim \mathcal{M}(H(n,1)) = n$ *and*

$$\dim \mathcal{M}(H(n,m)) = \binom{mn}{n} - 1 \qquad (m \geq 2).$$

Theorem 1.2 ([5]). *Let A and B be two finite dimensional n-Lie algebras. Then*

$$\dim \mathcal{M}(A \oplus B) = \dim \mathcal{M}(A) + \dim \mathcal{M}(B) + \binom{a+b}{n} - \binom{a}{n} - \binom{b}{n},$$

where $a = \dim A/A^2$ and $b = \dim B/B^2$.

Theorem 1.3 ([8]). *Let A be a finite dimensional n-Lie algebra and K be a central ideal of A. Then*

$$\dim \mathcal{M}(A) + \dim(A^2 \cap K) \leq \dim \mathcal{M}\left(\frac{A}{K}\right) + \dim \mathcal{M}(K) + a\binom{b}{n-1},$$

where $a = \dim K$ and $b = \dim(A/K)/(A/K)^2$.

Theorem 1.4 ([8]). *Let A be a d-dimensional nilpotent n-Lie algebra and $\dim A^2 = m \geq 1$. Then*

$$\dim \mathcal{M}(A) \leq \binom{d-m+1}{n} + (m-2)\binom{d-m}{n-1} + n - m.$$

2 Main results

For every d-dimensional nilpotent n-Lie algebra A, there exists two non-negative integers $t(A)$ and $s(A)$ such that $t(A) = \binom{d}{n} - \dim \mathcal{M}(A)$ and $s(A) = \binom{d-1}{n} + n - 1 - \dim \mathcal{M}(A)$. Thus

$$t(A) = \binom{d-1}{n-1} - n + 1 + s(A). \tag{1}$$

The classification of low dimensional Lie algebras is one of the fundamental issues in Lie algebras theory. The classification of Lie algebras can be found in many books and papers. The classification of the six-dimensional nilpotent Lie algebras on the arbitrary field was shown by Cicalo et al. [2]. Non-abelian nilpotent Lie algebras up to dimension five over an arbitrary field are $L_{3,2}, L_{4,2}, L_{4,3}$ and $L_{5,i}$ with $2 \leq i \leq 9$.

Lemma 2.1 ([3]). *The dimension of Schur multiplier of* $3, 4, 5$*-dimensional non-abelian nilpotent Lie algebras are as follows:*

(i) $\dim \mathcal{M}(L_{3,2}) = 2$, $\dim \mathcal{M}(L_{4,2}) = 4$ *and* $\dim \mathcal{M}(L_{4,3}) = 2$,

(ii) $\dim \mathcal{M}(L_{5,2}) = 7$, $\dim \mathcal{M}(L_{5,8}) = 6$ *and* $\dim \mathcal{M}(L_{5,4}) = 5$,

(iii) $\dim \mathcal{M}(L_{5,6}) = \dim \mathcal{M}(L_{5,7}) = \dim \mathcal{M}(L_{5,9}) = 3$, *and* $\dim \mathcal{M}(L_{5,3}) = \dim \mathcal{M}(L_{5,5}) = 4$.

Lemma 2.2 ([3])**.** *The dimension of Schur multiplier of* 6*-dimensional non-abelian nilpotent Lie algebras over a field of characteristic different from* 2*, are as follows:*

(i) $\dim \mathcal{M}(L_{5,2} \oplus F) = 11$, $\dim \mathcal{M}(L_{5,4} \oplus F) = \dim \mathcal{M}(L_{5,8} \oplus F) = 9$, $\dim \mathcal{M}(L_{5,3} \oplus F) = \dim \mathcal{M}(L_{5,5} \oplus F) = 7$, $\dim \mathcal{M}(L_{5,6} \oplus F) = \dim \mathcal{M}(L_{5,7} \oplus F) = \dim \mathcal{M}(L_{5,9} \oplus F) = 5$,

(ii) $\dim \mathcal{M}(L_{6,14}) = \dim \mathcal{M}(L_{6,16}) = 2$,

(iii) $\dim \mathcal{M}(L_{6,15}) = \dim \mathcal{M}(L_{6,17}) = \dim \mathcal{M}(L_{6,18}) = 3$,

(iv) $\dim \mathcal{M}(L_{6,13}) = \dim \mathcal{M}(L_{6,28}) = \dim \mathcal{M}(L_{6,21}(\epsilon)) = 4$,

(v) $\dim \mathcal{M}(L_{6,11}) = \dim \mathcal{M}(L_{6,12}) = \dim \mathcal{M}(L_{6,20}) = \dim \mathcal{M}(L_{6,19}(\epsilon)) = \dim \mathcal{M}(L_{6,24}(\epsilon)) = 5$,

(vi) $\dim \mathcal{M}(L_{6,10}) = \dim \mathcal{M}(L_{6,23}) = \dim \mathcal{M}(L_{6,25}) = \dim \mathcal{M}(L_{6,27}) = 6$,

(vii) $\dim \mathcal{M}(L_{6,26}) = \dim \mathcal{M}(L_{6,22}(\epsilon)) = 8$.

Nilpotent n-Lie algebras up to dimension $n+3$ over an arbitrary field, are well-known. There is just one non-abelian nilpotent n-Lie algebra with dimension $n+1$. This algebra is $H(n,1)$. There is two non-abelian nilpotent n-Lie algebras with dimension $n+2$. This algebras are $H(n,1) \oplus F(1)$ and $A_{n,n+2,1}$. There is seven non-abelian nilpotent n-Lie algebras with dimension $n+3$ when $n > 2$. This algebras are $A_{n,n+2,1} \oplus F(1)$, $H(n,1) \oplus F(2)$ and $A_{n,n+3,i}$ with $1 \leq i \leq 5$.

The value of $s(A)$ for $n+1$,$n+2$,$n+3$-dimensional non-abelian nilpotent n-Lie algebras is as following lemma.

Lemma 2.3. *(i)* $s(H(n,1)) = s(H(n,1) \oplus F(1)) = 0$ *and* $s(A_{n,n+2,1}) = n$.

(ii) $s(A_{n,n+2,1} \oplus F(1)) = s(A_{n,n+3,2}) = \binom{n+2}{n} - n - 1$, $s(H(n,1) \oplus F(2)) = 0$,

(iii) $s(A_{n,n+3,1}) = \binom{n+2}{n} - 3n + 1$, $s(A_{n,n+3,4}) = \binom{n+2}{n} - n$, $s(A_{n,n+3,5}) = s(A_{n,n+3,3}) = \binom{n+2}{n} - 2$.

Proof. (i) By Theorem 3.3 of [6], we have $\dim \mathcal{M}(H(n,1)) = \dim \mathcal{M}(A_{n,n+2,1}) = n$ and $\dim \mathcal{M}(H(n,1) \oplus F(1)) = 2n$. By direct calculation the proof is correct.

(ii) According to Lemma 4 of [3] and definition of $s(A)$, the proof is obvious. □

The notations of nilpotent Lie algebras in this paper are same as used in [2, 11].

Darabi et al. [6] classified all nilpotent n-Lie algebras A for which $s(A) = 0, 1, 2$ and applying it in order to determine all nilpotent n-Lie algebras A satisfying $0 \leq t(A) \leq 8$. Moreover, all nilpotent n-Lie algebras A for which $s(A) = 3$ are classified in [7].

Lemma 2.4 ([6, 7])**.** *Suppose A is a d-dimensional non-abelian nilpotent n-Lie algebra. Then*

(1) $s(A) = 0$ *if and only if* $A \cong H(n,1) \oplus F(d-n-1)$.

(2) $s(A) = 1$ *if and only if* $A \cong L_{5,8}$.

(3) $s(A) = 2$ *if and only if A is isomorphic to* $L_{4,3}$, $A_{3,6,1}$, $L_{5,8} \oplus F(1)$ *or* $H(2,t) \oplus F(d-2t-1)$ *for some* $t \geq 2$.

(4) $s(A) = 3$ *if and only if A is isomorphic to* $A_{3,5,2}$,$L_{5,3}$,$L_{5,5}$,$L_{6,22}(\epsilon)$,$L_{6,26}$,$L_{5,8} \oplus F(2)$,*or* $H(3,t) \oplus F(d-3t-1)$ *for some* $t \geq 2$.

Lemma 2.5. *Let A be a non-abelian d-dimensional nilpotent n-Lie algebra. Then*

(i) $s(A) = 4$ *if and only if A is isomorphic to one of the following Lie algebras* $L_{5,8} \oplus F(3)$, $L_{4,3} \oplus F(2)$, $L_{5,5} \oplus F(1)$, $L_{5,6}$, $L_{5,7}$, $L_{5,9}$,$L_{6,22}(\epsilon) \oplus F(1)$, $37A$, $H(4,r) \oplus F(d-4r-1)$ $(r \geq 2)$, $A_{4,6,1}$ *or* $A_{4,7,1}$.

(ii) $s(A) = 5$ *if and only if* A *is isomorphic to one of the following Lie algebras* $L_{5,8} \oplus F(4)$, $L_{4,3} \oplus F(3)$, $L_{5,5} \oplus F(2)$, $L_{6,22}(\epsilon) \oplus F(2)$, $L_{6,26} \oplus F(1)$, $L_{6,10}$, $L_{6,23}$, $L_{6,25}$, $L_{6,27}$, $37B$, $37D$, $H(5,r) \oplus F(d-5r-1)$ $(r \geq 2)$ *or* $A_{5,7,1}$.

The following Lemma can be found immediately by [15].

Lemma 2.6. *Let* A *be a non-abelian* d*-dimensional nilpotent Lie algebra. Then*

(i) *The only* 7*-dimensional nilpotent Lie algebras with* $S(A) = 6$ *are* $L_{6,10} \oplus F(1)$, $27A$ *and* 157.

(ii) *The only* 8*-dimensional nilpotent Lie algebras with* $S(A) = 6$ *are* $L_{4,3} \oplus F(4)$, $L_{5,5} \oplus F(3)$ *and* $37A \oplus F(1)$.

(iii) *The only* 7*-dimensional nilpotent Lie algebras with* $S(A) = 7$ *are* $27B$, $L_{6,23} \oplus F(1)$, $L_{6,25} \oplus F(1)$, $257A$, $257C$ *and* $257F$.

Now we state and prove our main theorems.

Theorem 2.7. *Suppose* A *is a* d*-dimensional non-abelian nilpotent* n*-Lie algebra with* $n > 2$. *Then*

(1) $t(A) = 17$ *if and only if* $A \cong A_{3,6,5}$.

(2) $t(A) = 18$ *if and only if* A *is isomorphic to* $H(4,1) \oplus F(1)$ *or* $A_{4,6,2}$.

Proof. We classified nilpotent n-Lie algebras with $n > 2$ and $t(A) = 17, 18$. Let A be a nilpotent n-Lie algbera $(n > 2)$. In the following cases, by relation (1), $s(A)$ in negative.

(i) $n \geq 4$ and $\dim A \geq n + 4$,

(ii) $n = 3$ and $\dim A \geq 8$.

Let $n = 3$ and $\dim A = 7$. If $t(A) = 17$, then by relation (1), $s(A) = 4$. By Lemma 2.5, the only algebra satisfying this condition is $A_{4,7,1}$. Similarly, if $t(A) = 18$, by relation (1), $s(A) = 5$. By Lemma 2.5, the only algebra satisfying this condition is $A_{5,7,1}$. For the rest of the proof, it is enough to search among d-dimensional nilpotent n-Lie algebras with $d \leq n + 3$. By lemmas 2.3, the only nilpotent n-Lie algebras satisfying this condition are $A_{17,19,1}$ and $A_{18,20,1}$ with $t(A) = 17$ and $t(A) = 18$, respectively. □

The following theorem characterize nilpotent Lie algebras with $t(A) = 11, 12$.

Theorem 2.8. *Suppose* A *is a nilpotent Lie algebra. Then*

(i) $t(A) = 11$ *if and only if* A *is isomorphic to* $H(2,1) \oplus F(10)$, $H(2,2) \oplus F(6)$, $H(2,3) \oplus F(4)$, $H(2,4) \oplus F(2)$, $H(2,5)$, $L_{6,22}(\epsilon) \oplus F(2)$, $L_{6,10} \oplus F(1)$, $27A$, 157, $L_{6,13}$, $L_{6,28}$ *or* $L_{6,21}(\epsilon)$.

(ii) $t(A) = 12$ *if and only if* A *is isomorphic to* $H(2,1) \oplus F(11)$, $H(2,2) \oplus F(7)$, $H(2,3) \oplus F(5)$, $H(2,4) \oplus F(3)$, $H(2,5) \oplus F(1)$, $L_{5,8} \oplus F(4)$, $L_{4,3} \oplus F(4)$, $L_{5,5} \oplus F(3)$, $37A \oplus F(3)$, $27B$, $L_{6,23} \oplus F(1)$, $L_{6,25} \oplus F(1)$, $257A$, $257C$, $257F$, $L_{6,15}$, $L_{6,17}$ *or* $L_{6,18}$.

Proof. (i) Let A be a d-dimensional nilpotent Lie algebra with $t(A) = 11$. By relation (1), we have $13 = d + s(A)$. By lemma 2.1, there is no d-dimensional nilpotent Lie algebra with $t(A) = 11$ for $d \leq 5$. According to lemma 2.2, the only 6-dimensional nilpotent Lie algebras with $t(A) = 11$ are $L_{6,13}$,$L_{6,28}$ and $L_{6,21}(\epsilon)$. For 7-dimensional nilpotent Lie algebra A with $t(A) = 11$, we have $s(A) = 6$. By lemma 2.6, the only 7-dimensional nilpotent Lie algebras with $t(A) = 11$ are $L_{6,10} \oplus F(1)$, $27A$ and 157.

For 8-dimensional and 9-dimensional nilpotent Lie algebra A with $t(A) = 11$, we have $s(A) = 5$ and $s(A) = 4$ respectively. By lemma 2.5, the only 8-dimensional nilpotent Lie algebra with $t(A) = 11$ are $L_{6,22}(\epsilon) \oplus F(2)$. There is no 9-dimensional nilpotent n-Lie algebra with $t(A) = 11$.

Similarly, nilpotent Lie algebras with $t(A) = 11$ of dimension $10, 11, 12$ and 13 are algebras with $s(A) = 3, 2, 1$ and 0, respectively. By Theorem 2.4, the following algebras are obtained:

$$H(2,1) \oplus F(10), H(2,2) \oplus F(6), H(2,3) \oplus F(4), H(2,4) \oplus F(4), H(2,5).$$

Since $s(A) \geq 0$, the proof is complete.

(ii) Let A be a d-dimensional nilpotent Lie algebra with $t(A) = 12$. By relation (1), we have $14 = d+s(A)$. Similarly case (i), there is no d-dimensional nilpotent Lie algebra with $t(A) = 12$ if $d \leq 5$. The only 6-dimensional nilpotent Lie algebras with $t(A) = 12$ are $L_{6,15}$, $L_{6,17}$ and $L_{6,18}$ by lemma 2.2.

For the 7-dimensional nilpotent Lie algebra A with $t(A) = 12$, we have $s(A) = 7$. By lemma 2.6, the only 7-dimensional nilpotent Lie algebras with $t(A) = 12$ are $27B$, $L_{6,23} \oplus F(1)$, $L_{6,25} \oplus F(1)$, $257A$, $257C$ and $257F$.

For the 8-dimensional nilpotent Lie algebra A with $t(A) = 12$, we have $s(A) = 6$. By lemma 2.6, the only 8-dimensional nilpotent Lie algebras with $t(A) = 12$ are $L_{4,3} \oplus F(4)$, $L_{5,5} \oplus F(3)$ and $37A \oplus F(1)$.

By lemma 2.5, the only 9-dimensional nilpotent Lie algebra with $t(A) = 12$ are $L_{5,8} \oplus F(4)$. There is no 10-dimensional nilpotent n-Lie algebra with $t(A) = 12$.

Similarly, nilpotent Lie algebras with $t(A) = 12$ of dimension $11, 12, 13$ and 14 are algebras with $s(A) = 3, 2, 1$ and 0, respectively. By lemma 2.4, the following algebras are obtained.

$$H(2,1) \oplus F(11), H(2,2) \oplus F(7), H(2,3) \oplus F(5), H(2,4) \oplus F(3), H(2,5) \oplus F(1).$$

As $s(A) \geq 0$, this completes the proof. □

We have listed the obtained n-Lie algebras in this paper in Table 1.

Table 1: all n-Lie algebras are obtained in this paper.

Name	Non-zero multiplication
$A_{n,n+2,1}$	$[e_1,\dots,e_n] = e_{n+1}, [e_2,\dots,e_{n+1}] = e_{n+2}$
$A_{n,n+3,1}$	$[e_1,\dots,e_n] = e_{n+1}, [e_2,\dots,e_n,e_{n+2}] = e_{n+3}$
$A_{n,n+3,2}$	$[e_1,\dots,e_n] = e_{n+1}, [e_2,\dots,e_{n+1}] = [e_1,e_3,\dots,e_n,e_{n+2}] = e_{n+3}$
$A_{n,n+3,3}$	$[e_1,\dots,e_n] = e_{n+1}, [e_2,\dots,e_{n+1}] = e_{n+2},\ [e_1,e_3,\dots,e_{n+1}] = e_{n+3}$
$A_{n,n+3,4}$	$[e_1,\dots,e_n] = e_{n+1}, [e_2,\dots,e_{n+1}] = e_{n+2},\ [e_2,\dots,e_n,e_{n+2}] = e_{n+3}$
$A_{n,n+3,5}$	$[e_1,\dots,e_n] = e_{n+1}, [e_2,\dots,e_{n+1}] = e_{n+2}$, $[e_2,\dots,e_n,e_{n+2}] = [e_1,e_3,\dots,e_{n+1}] = e_{n+3}$
$L_{4,3}$	$[e_1,e_2] = e_3,\ [e_1,e_3] = e_4$
$L_{5,5}$	$[e_1,e_2] = e_3,\ [e_1,e_3] = [e_2,e_4] = e_5$
$L_{5,8}$	$[e_1,e_2] = e_4,\ [e_1,e_3] = e_5$
$L_{6,10}$	$[e_1,e_2] = e_3, [e_1,e_3] = [e_4,e_5] = e_6$
$L_{6,13}$	$[e_1,e_2] = e_3, [e_1,e_3] = [e_2,e_4] = e_5, [e_1,e_5] = [e_3,e_4] = e_6$
$L_{6,15}$	$[e_1,e_2] = e_3, [e_1,e_3] = e_4, [e_1,e_4] = [e_2,e_3] = e_5, [e_1,e_5] = [e_2,e_4] = e_6$
$L_{6,17}$	$[e_1,e_2] = e_3, [e_1,e_3] = e_4, [e_1,e_4] = e_5, [e_1,e_5] = [e_2,e_3] = e_6$
$L_{6,18}$	$[e_1,e_2] = e_3, [e_1,e_3] = e_4, [e_1,e_4] = e_5, [e_1,e_5] = e_6$
$L_{6,21}(\epsilon)$	$[e_1,e_2] = e_3, [e_1,e_3] = e_4, [e_2,e_3] = e_5, [e_1,e_4] = e_6,\ [e_2,e_5] = \epsilon e_6$
$L_{6,22}(\epsilon)$	$[e_1,e_2] = [e_3,e_4] = e_5, [e_1,e_3] = e_6, [e_2,e_4] = \epsilon e_6$
$L_{6,23}$	$[e_1,e_2] = e_3, [e_1,e_3] = [e_2,e_4] = e_5, [e_1,e_4] = e_6$
$L_{6,25}$	$[e_1,e_2] = e_3, [e_1,e_3] = e_5, [e_1,e_4] = e_6$
$L_{6,28}$	$[e_1,e_2] = e_3, [e_1,e_3] = e_4, [e_1,e_4] = e_5, [e_2,e_3] = e_6$
$27A$	$[e_1,e_2] = e_6, [e_1,e_4] = [e_3,e_5] = e_7$
$27B$	$[e_1,e_2] = [e_3,e_4] = e_6, [e_1,e_5] = [e_2,e_3] = e_7$
157	$[e_1,e_2] = e_3, [e_1,e_3] = [e_2,e_4] = [e_5,e_6] = e_7$
$37A$	$[e_1,e_2] = e_5, [e_2,e_3] = e_6, [e_2,e_4] = e_7$
$257A$	$[e_1,e_2] = e_3, [e_1,e_3] = [e_2,e_4] = e_6, [e_1,e_5] = e_7$
$257C$	$[e_1,e_2] = e_3, [e_1,e_3] = [e_2,e_4] = e_6, [e_2,e_5] = e_7$
$257F$	$[e_1,e_2] = e_3, [e_2,e_3] = [e_4,e_5] = e_6, [e_2,e_4] = e_7$

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